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# Chapter 5

# *Some Types of HyperNeutrosophic Set (3): Dynamic, Quadripartitioned, Pentapartitioned, Heptapartitioned, m-polar*

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## Abstract

This paper builds upon the foundation established in [50, 51]. The Neutrosophic Set provides a robust mathematical framework for handling uncertainty, defined by three membership functions: truth, indeterminacy, and falsity. Recent developments have introduced extensions such as the Hyperneutrosophic Set and SuperHyperneutrosophic Set to tackle increasingly complex and multidimensional problems.

In this study, we explore further extensions, including the Dynamic Neutrosophic Set, Quadripartitioned Neutrosophic Set, Pentapartitioned Neutrosophic Set, Heptapartitioned Neutrosophic Set, and m-Polar Neutrosophic Set, to address advanced challenges and applications.

*Keywords:* Set Theory, SuperhyperNeutrosophic set, Neutrosophic Set, HyperNeutrosophic set

## 1 Preliminaries and Definitions

This section introduces the key concepts and definitions fundamental to the discussions in this paper. The study employs standard set-theoretic operations and extends them to advanced frameworks. Readers seeking an in-depth understanding of classical set theory are referred to resources such as [25, 68, 71, 72]. For foundational principles and applications of Neutrosophic Sets, the referenced literature provides comprehensive insights.

### 1.1 Neutrosophic, HyperNeutrosophic, and n-SuperHyperNeutrosophic Sets

Various set-theoretic frameworks have been devised to address uncertainty, vagueness, and imprecision in decision-making. These frameworks include Fuzzy Sets [120–124], Intuitionistic Fuzzy Sets [6–11], Vague Sets [2, 12, 64, 73, 97], Plithogenic Sets [27, 34, 37–39, 47, 48, 60, 109, 111, 112], Soft Sets [65, 74, 79], Hypersoft Sets [30, 42, 59, 110], and Neutrosophic Sets [31, 32, 35, 46, 52–57, 61, 62, 106, 107, 114].

Neutrosophic Sets extend the concept of Fuzzy Sets by incorporating a third dimension—indeterminacy—alongside truth and falsity [104–107]. This approach enables a more nuanced representation of uncertainty and ambiguity. Further advancements have resulted in the development of HyperNeutrosophic Sets and n-SuperHyperNeutrosophic Sets, which are tailored for addressing more intricate and high-dimensional problems [29, 41].

The following subsections present precise definitions and critical attributes of these extended frameworks.

**Definition 1.1** (Base Set)**.** A *base set* $S$ is the foundational set from which complex structures such as powersets and hyperstructures are derived. It is formally defined as:

$$S = \{x \mid x \text{ is an element within a specified domain}\}.$$

All elements in constructs like $\mathcal{P}(S)$ or $\mathcal{P}_n(S)$ originate from the elements of $S$.

**Definition 1.2** (Powerset)**.** [37, 94] The *powerset* of a set $S$, denoted $\mathcal{P}(S)$, is the collection of all possible subsets of $S$, including both the empty set and $S$ itself. Formally, it is expressed as:

$$\mathcal{P}(S) = \{A \mid A \subseteq S\}.$$

**Definition 1.3** ($n$-th Powerset)**.** (cf. [26, 37, 43, 103, 113])

The $n$-th powerset of a set $H$, denoted $P_n(H)$, is defined iteratively, starting with the standard powerset. The recursive construction is given by:

$$P_1(H) = P(H), \quad P_{n+1}(H) = P(P_n(H)), \quad \text{for } n \geq 1.$$

Similarly, the $n$-th non-empty powerset, denoted $P_n^*(H)$, is defined recursively as:

$$P_1^*(H) = P^*(H), \quad P_{n+1}^*(H) = P^*(P_n^*(H)).$$

Here, $P^*(H)$ represents the powerset of $H$ with the empty set removed.

**Definition 1.4** (Neutrosophic Set)**.** [106, 107] Let $X$ be a non-empty set. A *Neutrosophic Set (NS) $A$* on $X$ is characterized by three membership functions:

$$T_A : X \to [0, 1], \quad I_A : X \to [0, 1], \quad F_A : X \to [0, 1],$$

where for each $x \in X$, the values $T_A(x)$, $I_A(x)$, and $F_A(x)$ represent the degrees of truth, indeterminacy, and falsity, respectively. These values satisfy the following condition:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

**Definition 1.5** (HyperNeutrosophic Set)**.** (cf. [29, 41, 44, 45, 108]) Let $X$ be a non-empty set. A *HyperNeutrosophic Set (HNS) $\tilde{A}$* on $X$ is a mapping:

$$\tilde{\mu} : X \to \mathcal{P}([0, 1]^3),$$

where $\mathcal{P}([0, 1]^3)$ is the family of all non-empty subsets of the unit cube $[0, 1]^3$. For each $x \in X$, $\tilde{\mu}(x) \subseteq [0, 1]^3$ is a set of neutrosophic membership triplets $(T, I, F)$ that satisfy:

$$0 \leq T + I + F \leq 3.$$

**Definition 1.6** ($n$-SuperHyperNeutrosophic Set)**.** (cf. [29, 41, 44, 45]) Let $X$ be a non-empty set. An *$n$-SuperHyperNeutrosophic Set (n-SHNS)* is a recursive generalization of Neutrosophic Sets and HyperNeutrosophic Sets. It is defined as a mapping:

$$\tilde{A}_n : \mathcal{P}_n(X) \to \mathcal{P}_n([0, 1]^3),$$

where:

- $\mathcal{P}_1(X) = \mathcal{P}(X)$, the power set of $X$, and for $k \geq 2$,

  $$\mathcal{P}_k(X) = \mathcal{P}(\mathcal{P}_{k-1}(X)),$$

  representing the $k$-th nested family of non-empty subsets of $X$.
- $\mathcal{P}_n([0, 1]^3)$ is defined similarly for the unit cube $[0, 1]^3$.

For each $A \in \mathcal{P}_n(X)$ and $(T, I, F) \in \tilde{A}_n(A)$, the following condition is satisfied:

$$0 \leq T + I + F \leq 3,$$

where $T, I, F$ represent the degrees of truth, indeterminacy, and falsity for the $n$-th level subsets of $X$.

## 2 Results of This Paper

This section outlines the main results presented in this paper.

## 2.1 Dymanic HyperNeutrosophic set

A Dynamic Neutrosophic Set incorporates time-dependent truth, indeterminacy, and falsity functions, evolving continuously within a time domain [19, 84, 96, 115–117, 119]. This is extended using Hyperneutrosophic Sets and SuperHyperneutrosophic Sets.

**Definition 2.1** (Dynamic Neutrosophic Set)**.** (cf. [116]) Let $U$ be a universal set, and $A \subseteq U$ be a neutrosophic set. A *Dynamic Neutrosophic Set (DNS)* is defined with respect to a time parameter $t$ (where $t \in T, T \subseteq \mathbb{R}^+$) as:

$$D_A^t = \{\langle x, T_A(t), I_A(t), F_A(t)\rangle \mid x \in U\},$$

where:

- $T_A(t) : U \to [0,1]$ is the *truth-membership function* at time $t$,
- $I_A(t) : U \to [0,1]$ is the *indeterminacy-membership function* at time $t$,
- $F_A(t) : U \to [0,1]$ is the *falsity-membership function* at time $t$,

such that for all $x \in U$,

$$T_A(t) + I_A(t) + F_A(t) \le 1.$$

The functions $T_A(t), I_A(t), F_A(t)$ are time-dependent and continuous over $T$. The evolution of the neutrosophic components is represented as:

$$T_A(t), I_A(t), F_A(t) : T \to [0,1].$$

**Definition 2.2** (Dynamic Hyperneutrosophic Set (DHNS))**.** Let $X$ be a non-empty set, and let $T \subseteq \mathbb{R}^+$ be a time domain. A *Dynamic Hyperneutrosophic Set* (DHNS) $\widetilde{D}$ on $X$ is specified by a mapping:

$$\widetilde{D} : X \times T \;\to\; \mathcal{P}([0,1]^3),$$

such that for each $(x,t) \in X \times T$,

$$\widetilde{D}(x,t) \;\subseteq\; \{(T,I,F) \in [0,1]^3 : T + I + F \le 3\}.$$

Equivalently, for each $t \in T$, the function

$$\widetilde{D}_t : X \to \mathcal{P}([0,1]^3), \quad \widetilde{D}_t(x) := \widetilde{D}(x,t),$$

is a Hyperneutrosophic Set in the usual sense, but one that evolves over time $t$. Each $\widetilde{D}_t$ is presumably continuous in $t$ in some sense (optional constraint), reflecting how membership sets might change as $t$ progresses.

**Theorem 2.3.** *Every Dynamic Neutrosophic Set is a special case of a Dynamic Hyperneutrosophic Set.*

*Proof.* A *Dynamic Neutrosophic Set (DNS)* $\mathcal{D}$ on $U$ (Definition **??**) assigns each $(x,t)$ a single triplet $(T_A(t)(x), I_A(t)(x), F_A(t)(x))$. Let

$$\widetilde{D}(x,t) = \Big\{\big(T_A(t)(x),\ I_A(t)(x),\ F_A(t)(x)\big)\Big\}.$$

Thus, each $(x,t)$ maps to a *singleton* in $[0,1]^3$. We see that $\widetilde{D} : X \times T \to \mathcal{P}([0,1]^3)$ is a Dynamic Hyperneutrosophic Set. The only difference is that $\widetilde{D}(x,t)$ remains single-valued. Hence, every DNS is embedded in DHNS as a degenerate (singleton) membership set for each $(x,t)$. □

**Theorem 2.4.** *Every Hyperneutrosophic Set is a special case of a Dynamic Hyperneutrosophic Set by ignoring time dependence or taking T to be a singleton domain.*

*Proof.* A *Hyperneutrosophic Set (HNS)* $\tilde{A}$ is a mapping $X \to \mathcal{P}([0,1]^3)$. In Definition 2.2, we have $\widetilde{D}$ : $X \times T \to \mathcal{P}([0,1]^3)$. If we fix $t = t_0$ or let $T = \{t_0\}$ be a single point in time, then

$$\widetilde{D}(x,t_0) = \tilde{A}(x).$$

Hence, ignoring or collapsing the time axis recovers the standard hyperneutrosophic membership. Thus, HNS is a sub-case of DHNS where time is trivial or absent. □

**Definition 2.5** (Dynamic $n$-SuperHyperneutrosophic Set (D-$n$-SHNS))**.** Let $X$ be a non-empty set and $T \subseteq \mathbb{R}^+$ be the time domain. Define

$$\mathcal{P}_1(X) = \mathcal{P}(X), \quad \mathcal{P}_k(X) = \mathcal{P}\big(\mathcal{P}_{k-1}(X)\big) \quad (k \geq 2).$$

Similarly, let $\mathcal{P}_n\big([0,1]^3\big)$ represent the $n$-th nested family of non-empty subsets of $[0,1]^3$. A *Dynamic n-SuperHyperneutrosophic Set (D-n-SHNS)* is a mapping:

$$\widetilde{D}_n : \mathcal{P}_n(X) \times T \longrightarrow \mathcal{P}_n\big([0,1]^3\big),$$

such that for each $(A, t) \in \mathcal{P}_n(X) \times T$:

$$\widetilde{D}_n(A, t) \subseteq \big\{(T, I, F) \in [0,1]^3 : T + I + F \leq 3\big\}.$$

That is, each $n$-th level subset $A$ is assigned a *set* of membership triplets $(T, I, F)$, each lying in $[0,1]^3$ with $T + I + F \leq 3$, and these sets vary over the time parameter $t$.

**Theorem 2.6.** *Every Dynamic Hyperneutrosophic Set is a special case of a Dynamic n-SuperHyperneutrosophic Set (D-n-SHNS) for $n = 1$.*

*Proof.* A *Dynamic Hyperneutrosophic Set (DHNS) $\widetilde{D}$* is a mapping $X \times T \to \mathcal{P}([0,1]^3)$. In Definition 2.5, set $n = 1$. Then $\mathcal{P}_1(X) = \mathcal{P}(X)$, and we define

$$\widetilde{D}_1(A, t) = \begin{cases} \widetilde{D}(x, t), & \text{if } A = \{x\}, \\ \varnothing, & \text{otherwise.} \end{cases}$$

Hence, for singletons $A = \{x\}$, we recover exactly the membership sets from $\widetilde{D}(x, t)$. Therefore, a DHNS is embedded in D-1-SHNS as a single-level case. □

**Theorem 2.7.** *Every (static) n-SuperHyperneutrosophic Set is a special case of a Dynamic n-SuperHyperneutrosophic Set by taking $T$ to be a single point or ignoring time.*

*Proof.* An *n-SuperHyperneutrosophic Set (SHNS) $\tilde{A}_n$* is a mapping $\mathcal{P}_n(X) \to \mathcal{P}_n([0,1]^3)$. In Definition 2.5, we have $\widetilde{D}_n : \mathcal{P}_n(X) \times T \to \mathcal{P}_n([0,1]^3)$. If we let $T = \{t_0\}$ be a single point in time (or otherwise disregard time), we can define

$$\widetilde{D}_n(A, t_0) = \tilde{A}_n(A).$$

Then ignoring the time dimension yields the standard $n$-SuperHyperneutrosophic membership sets. Thus, every $n$-SHNS is included in D-$n$-SHNS by collapsing $T$ to a singleton or skipping time dependence. □

### 2.2 Hyper Quadripartitioned Neutrosophic set

A Quadripartitioned Neutrosophic Set assigns four membership values [16,17,23,49,69,70,86,88,99–101,104]. This is extended using Hyperneutrosophic Sets and SuperHyperneutrosophic Sets.

**Definition 2.8** (Quadripartitioned Neutrosophic Set (QNS))**.** (cf. [16,23,69]) Let $X$ be a universe of discourse. A *Quadripartitioned Neutrosophic Set (QNS)* on $X$ is given by

$$QNS = \{\langle x, T(x), C(x), U(x), F(x)\rangle \mid x \in X\},$$

where each of $T(x), C(x), U(x), F(x)$ lies in $[0,1]$, satisfying

$$0 \leq T(x) + C(x) + U(x) + F(x) \leq 4.$$

**Definition 2.9** (Hyper Quadripartitioned Neutrosophic Set (HQNS))**.** Let $X$ be a non-empty set, and consider the family $\mathcal{P}([0,1]^4)$ of all non-empty subsets of the unit 4-cube $[0,1]^4$. A *Hyper Quadripartitioned Neutrosophic Set (HQNS) $\widetilde{Q}$* on $X$ is a mapping

$$\widetilde{Q} : X \longrightarrow \mathcal{P}([0,1]^4),$$

such that for each $x \in X$,

$$\widetilde{Q}(x) \subseteq \Big\{(T, C, U, F) \in [0,1]^4 : T + C + U + F \leq 4\Big\}.$$

That is, each point $x \in X$ is assigned a *set* of quadripartitioned membership quadruples $\big(T, C, U, F\big)$, each lying in $[0,1]^4$ with $T + C + U + F \leq 4$.

**Theorem 2.10.** *Every Quadripartitioned Neutrosophic Set is a special case of a Hyper Quadripartitioned Neutrosophic Set.*

*Proof.* A *Quadripartitioned Neutrosophic Set (QNS) $Q$* assigns each $x \in X$ exactly one quadruple

$$(T(x), C(x), U(x), F(x)) \in [0, 1]^4$$

with $T + C + U + F \leq 4$. In Definition 2.9, we let

$$\widetilde{Q}(x) = \Big\{\big(T(x),\ C(x),\ U(x),\ F(x)\big)\Big\},$$

a *singleton* subset of $[0, 1]^4$. The same constraint $T + C + U + F \leq 4$ persists, so each single-valued QNS is embedded in the HQNS framework as a degenerate (singleton) membership set for each $x$. □

**Theorem 2.11.** *Every HyperNeutrosophic Set is a special case of a Hyper Quadripartitioned Neutrosophic Set by treating the four components as, for instance, $(T, I, 0, F)$ (or by ignoring $C$ and $U$).*

*Proof.* A *HyperNeutrosophic Set (HNS) $\tilde{A}$* maps each $x \in X$ to a subset of $[0, 1]^3$ with $T + I + F \leq 3$. In Definition 2.9, each membership is a subset of $[0, 1]^4$ with $T + C + U + F \leq 4$. If we fix $C = 0$ and $U = I$ (or $U = 0$) so that effectively $(T, C, U, F)$ becomes $(T, 0, I, F)$, and require $T + 0 + I + F = T + I + F \leq 3$, we see that ignoring or collapsing the extra dimension recovers an HNS membership set. Specifically, define

$$\widetilde{Q}(x) = \Big\{\big(T, I, 0, F\big) : (T, I, F) \in \tilde{A}(x)\Big\}.$$

Hence, ignoring two of the four components (or setting them to zero or merging them) yields the usual 3-component hyperneutrosophic membership. Therefore, an HNS is subsumed under HQNS by discarding or collapsing extra components to zero. □

**Definition 2.12** ($n$-SuperHyper Quadripartitioned Neutrosophic Set (n-SHQNS))**.** Let $X$ be a non-empty set. Define:

$$\mathcal{P}_1(X) = \mathcal{P}(X), \quad \mathcal{P}_k(X) = \mathcal{P}\big(\mathcal{P}_{k-1}(X)\big) \ \ (k \geq 2).$$

Similarly, let $\mathcal{P}_n\big([0, 1]^4\big)$ denote the $n$-th nested family of non-empty subsets of $[0, 1]^4$. A *n-SuperHyper Quadripartitioned Neutrosophic Set (n-SHQNS)* is a mapping

$$\widetilde{Q}_n : \mathcal{P}_n(X) \ \longrightarrow\ \mathcal{P}_n\big([0, 1]^4\big),$$

such that for each $A \in \mathcal{P}_n(X)$,

$$\widetilde{Q}_n(A)\ \subseteq\ \Big\{(T, C, U, F) \in [0, 1]^4 :\ T + C + U + F \leq 4\Big\}.$$

Hence, each $n$-th level subset $A \subseteq X$ is assigned a *set* of four-part membership quadruples $(T, C, U, F)$ in $[0, 1]^4$ with $T + C + U + F \leq 4$.

**Theorem 2.13.** *Every Hyper Quadripartitioned Neutrosophic Set is a special case of an n-SuperHyper Quadripartitioned Neutrosophic Set (n-SHQNS) for $n = 1$.*

*Proof.* A *Hyper Quadripartitioned Neutrosophic Set (HQNS) $\widetilde{Q}$* is a mapping $X \to \mathcal{P}([0, 1]^4)$ with $T + C + U + F \leq 4$. In Definition 2.12, set $n = 1$, so

$$\widetilde{Q}_1 : \mathcal{P}_1(X) = \mathcal{P}(X) \ \ \to\ \ \mathcal{P}_1\big([0, 1]^4\big) = \mathcal{P}([0, 1]^4).$$

Define

$$\widetilde{Q}_1(\{x\}) := \widetilde{Q}(x), \quad \widetilde{Q}_1(A) = \varnothing \quad \text{for } A \neq \{x\}.$$

Hence, for singletons $A = \{x\} \subseteq X$, we recover precisely $\widetilde{Q}(x)$. The constraint $T + C + U + F \leq 4$ remains the same, embedded in $\widetilde{Q}_1(A)$. Therefore, any HQNS is realized as a degenerate single-level mapping in an $n$-SHQNS with $n = 1$. □

**Theorem 2.14.** *Every n-SuperHyperNeutrosophic Set is a special case of an n-SuperHyper Quadripartitioned Neutrosophic Set by ignoring or collapsing the fourth component (e.g., $C = 0$ or $U = I$).*

*Proof.* An *n-SuperHyperNeutrosophic Set (SHNS)* $\tilde{A}_n$ maps each $A \in \mathcal{P}_n(X)$ to a set of $(T, I, F)$ in $[0, 1]^3$ with $T + I + F \leq 3$. In Definition 2.12, each membership is in $[0, 1]^4$ with $T + C + U + F \leq 4$. To retrieve an SHNS, fix or zero out some components. For instance, set $C = 0$, $U = I$, and require $T + I + F = T + (U) + F \leq 3$. Concretely, define

$$\widetilde{Q}_n(A) = \Big\{(T, 0, I, F) : (T, I, F) \in \tilde{A}_n(A)\Big\}.$$

Hence, ignoring the extra dimension(s) reverts membership to $(T, I, F) \in [0, 1]^3$. Therefore, an $n$-SHNS is a special case of n-SHQNS with additional constraints or identification of dimensions. □

### 2.3 Hyper Pentapartitioned Neutrosophic set

A Pentapartitioned Neutrosophic Set assigns five membership values [5, 13, 15, 20–22, 28, 33, 36, 40, 58, 76, 85, 87]. This is extended using Hyperneutrosophic Sets and SuperHyperneutrosophic Sets.

**Definition 2.15** (Pentapartitioned Neutrosophic Set (PNS))**.** (cf. [13, 76, 85]) Let $X$ be a universe of discourse. A *Pentapartitioned Neutrosophic Set (PNS)* on $X$ is given by

$$PNS = \{\langle x, T(x), C(x), R(x), U(x), F(x)\rangle \mid x \in X\},$$

where each of $T(x), C(x), R(x), U(x), F(x) \in [0, 1]$, satisfying

$$0 \;\leq\; T(x) + C(x) + R(x) + U(x) + F(x) \;\leq\; 5.$$

**Definition 2.16** (Hyper Pentapartitioned Neutrosophic Set (HPNS))**.** Let $X$ be a non-empty set, and let $\mathcal{P}([0, 1]^5)$ denote the family of all non-empty subsets of the unit 5-cube $[0, 1]^5$. A *Hyper Pentapartitioned Neutrosophic Set (HPNS)* $\widetilde{P}$ on $X$ is a mapping

$$\widetilde{P} : X \;\to\; \mathcal{P}([0, 1]^5),$$

such that for each $x \in X$,

$$\widetilde{P}(x) \;\subseteq\; \Big\{(T, C, R, U, F) \in [0, 1]^5 : T + C + R + U + F \;\leq\; 5\Big\}.$$

Hence, each $x \in X$ is assigned a *set* of pentapartitioned membership quintuples $\big(T, C, R, U, F\big)$, where $T + C + R + U + F \leq 5$.

**Theorem 2.17.** *Every Pentapartitioned Neutrosophic Set is a special case of a Hyper Pentapartitioned Neutrosophic Set.*

*Proof.* A *Pentapartitioned Neutrosophic Set (PNS)* assigns each $x \in X$ a unique quintuple

$$(T(x), C(x), R(x), U(x), F(x)) \in [0, 1]^5$$

satisfying $T + C + R + U + F \leq 5$. In Definition 2.16, we map each $x$ to a *set* of such quintuples. Define:

$$\widetilde{P}(x) = \Big\{\big(T(x),\; C(x),\; R(x),\; U(x),\; F(x)\big)\Big\},$$

i.e. a singleton set. The same constraint $T + C + R + U + F \leq 5$ persists. Consequently, each PNS is embedded in HPNS as a degenerate (singleton) membership set for each $x$. □

**Theorem 2.18.** *Every HyperNeutrosophic Set is a special case of a Hyper Pentapartitioned Neutrosophic Set by reducing two membership components to zero (e.g., $C = R = 0$).*

*Proof.* A *HyperNeutrosophic Set (HNS)* $\tilde{A}$ maps each $x \in X$ to a subset of $[0,1]^3$ (triplets $(T, I, F)$ with $T + I + F \leq 3$). In Definition 2.16, each membership is a subset of $[0,1]^5$ with $T + C + R + U + F \leq 5$. If we identify $(T, I, F)$ in $[0,1]^3$ with $(T, 0, 0, I, F)$ in $[0,1]^5$, we require $T + 0 + 0 + I + F = T + I + F \leq 3$. We can embed an HNS as:

$$\widetilde{P}(x) = \Big\{(T, 0, 0, I, F) : (T, I, F) \in \tilde{A}(x)\Big\}.$$

Hence, ignoring or zeroing out two membership parts ($C = 0$, $R = 0$) recovers a 3-part hyperneutrosophic membership. Therefore, an HNS is included in HPNS by discarding extra membership dimensions. □

**Definition 2.19** ($n$-SuperHyper Pentapartitioned Neutrosophic Set (n-SHPNS))**.** Let $X$ be a non-empty set. Define recursively:

$$\mathcal{P}_1(X) = \mathcal{P}(X), \quad \mathcal{P}_k(X) = \mathcal{P}\big(\mathcal{P}_{k-1}(X)\big) \quad \text{for } k \geq 2.$$

Similarly, let $\mathcal{P}_n([0,1]^5)$ denote the $n$-th nested family of non-empty subsets of $[0,1]^5$. A *n-SuperHyper Pentapartitioned Neutrosophic Set (n-SHPNS)* is a mapping

$$\widetilde{P}_n : \mathcal{P}_n(X) \longrightarrow \mathcal{P}_n\big([0,1]^5\big),$$

such that for each $A \in \mathcal{P}_n(X)$,

$$\widetilde{P}_n(A) \subseteq \Big\{(T, C, R, U, F) \in [0,1]^5 : T + C + R + U + F \leq 5\Big\}.$$

Hence, each $n$-th level subset $A$ is assigned a *set* of pentapartitioned membership quintuples $(T, C, R, U, F) \in [0,1]^5$, with $T + C + R + U + F \leq 5$.

**Theorem 2.20.** *Every Hyper Pentapartitioned Neutrosophic Set is a special case of an n-SuperHyper Pentapartitioned Neutrosophic Set (n-SHPNS) for $n = 1$.*

*Proof.* A *Hyper Pentapartitioned Neutrosophic Set (HPNS)* $\widetilde{P}$ is a mapping $X \to \mathcal{P}([0,1]^5)$ with $T + C + R + U + F \leq 5$. In Definition 2.19, set $n = 1$. Then

$$\widetilde{P}_1 : \mathcal{P}_1(X) = \mathcal{P}(X) \to \mathcal{P}_1\big([0,1]^5\big) = \mathcal{P}\big([0,1]^5\big).$$

Define

$$\widetilde{P}_1(\{x\}) := \widetilde{P}(x), \quad \widetilde{P}_1(A) = \varnothing \quad (\text{for } A \neq \{x\}).$$

Hence, for singletons $A = \{x\} \subseteq X$, we recover exactly $\widetilde{P}(x)$. The constraint $T + C + R + U + F \leq 5$ is maintained. Therefore, any HPNS is subsumed in n-SHPNS with $n = 1$. □

**Theorem 2.21.** *Every n-SuperHyperneutrosophic Set is a special case of an n-SuperHyper Pentapartitioned Neutrosophic Set by reducing two membership components to zero (e.g., $C = R = 0$).*

*Proof.* An *n-SuperHyperneutrosophic Set (SHNS)* $\tilde{A}_n$ maps each $A \in \mathcal{P}_n(X)$ to a subset of $[0,1]^3$ (triplets $(T, I, F)$). Compare with Definition 2.19, where $\widetilde{P}_n(A)$ is a subset of $[0,1]^5$ with $T + C + R + U + F \leq 5$. To recover an SHNS from n-SHPNS, we set $C = R = 0$ and identify $U = I$. Hence $(T, C, R, U, F) = (T, 0, 0, I, F)$ with $T + I + F \leq 3$. Formally, define

$$\widetilde{P}_n(A) = \Big\{(T, 0, 0, I, F) : (T, I, F) \in \tilde{A}_n(A)\Big\}.$$

Hence, ignoring or zeroing out two membership parts yields a standard 3-part membership in $[0,1]^3$. Thus, each SHNS is included in an n-SHPNS by discarding extra membership dimensions. □

### 2.4 Hyper Heptapartitioned Neutrosophic Set

A Heptapartitioned Neutrosophic Set assigns seven membership [14, 44, 80–82]. This is extended using Hyperneutrosophic Sets and SuperHyperneutrosophic Sets.

**Definition 2.22** (Heptapartitioned Neutrosophic Set)**.** [14, 81] A Heptapartitioned Neutrosophic Set (HNS) on a universe $X$ is defined as:

$$HNS = \{\langle x, T(x), C(x), R(x), U(x), F(x), G(x), L(x)\rangle \mid x \in X\},$$

where $T(x), C(x), R(x), U(x), F(x), G(x), L(x) \in [0,1]$, and

$$0 \leq T(x) + C(x) + R(x) + U(x) + F(x) + G(x) + L(x) \leq 7.$$

**Definition 2.23** (Hyper Heptapartitioned Neutrosophic Set (HHNS))**.** Let $X$ be a non-empty set, and let $\mathcal{P}([0,1]^7)$ denote the family of all non-empty subsets of the 7-cube $[0,1]^7$. A *Hyper Heptapartitioned Neutrosophic Set (HHNS)* $\widetilde{H}$ on $X$ is a mapping

$$\widetilde{H} : X \longrightarrow \mathcal{P}([0,1]^7),$$

such that for each $x \in X$,

$$\widetilde{H}(x) \subseteq \left\{ (T, C, R, U, F, G, L) \in [0,1]^7 : T + C + R + U + F + G + L \leq 7 \right\}.$$

Hence, every point $x \in X$ is assigned a *set* of heptapartitioned membership 7-tuples $(T, C, R, U, F, G, L)$, each lying in $[0,1]^7$ with $T + C + R + U + F + G + L \leq 7$.

**Theorem 2.24.** *Every Heptapartitioned Neutrosophic Set is a special case of a Hyper Heptapartitioned Neutrosophic Set.*

*Proof.* A *Heptapartitioned Neutrosophic Set (HptNS)* $H$ assigns each $x \in X$ exactly one 7-tuple $(T, C, R, U, F, G, L) \in [0,1]^7$ with $T + C + R + U + F + G + L \leq 7$. To embed this in Definition 2.23, define

$$\widetilde{H}(x) = \Big\{ \big(T(x),\ C(x),\ R(x),\ U(x),\ F(x),\ G(x),\ L(x)\big) \Big\},$$

i.e. a singleton set in $[0,1]^7$. Since the same $T+C+R+U+F+G+L \leq 7$ constraint remains, each single-valued HptNS is embedded in HHNS as a degenerate membership set (a singleton). □

**Theorem 2.25.** *Every HyperNeutrosophic Set is a special case of a Hyper Heptapartitioned Neutrosophic Set by ignoring four of the membership components.*

*Proof.* A *HyperNeutrosophic Set (HNS)* $\tilde{A}$ maps each $x \in X$ to a subset of $[0,1]^3$ (triplets $(T, I, F)$). In Definition 2.23, each membership is a subset of $[0,1]^7$ with $T + C + R + U + F + G + L \leq 7$. If we fix four components to zero (e.g., $C = R = G = L = 0$) and rename $U = I$, then $(T, I, F)$ in $[0,1]^3$ becomes $(T, 0, 0, I, F, 0, 0)$ in $[0,1]^7$, needing $T + 0 + 0 + I + F + 0 + 0 = T + I + F \leq 3$. Formally:

$$\widetilde{H}(x) = \Big\{ \big(T, 0, 0, I, F, 0, 0\big) : (T, I, F) \in \tilde{A}(x) \Big\}.$$

Hence, ignoring the extra membership components merges an HNS into an HHNS. Therefore, an HNS is a special case of HHNS by discarding (or zeroing out) the four additional partitions. □

**Definition 2.26** ($n$-SuperHyper Heptapartitioned Neutrosophic Set (n-SHHNS))**.** Let $X$ be a non-empty set. Define recursively:

$$\mathcal{P}_1(X) = \mathcal{P}(X), \quad \mathcal{P}_k(X) = \mathcal{P}\big(\mathcal{P}_{k-1}(X)\big) \quad (k \geq 2).$$

Similarly, let $\mathcal{P}_n\big([0,1]^7\big)$ denote the $n$-th nested family of non-empty subsets of the 7-cube $[0,1]^7$. A *$n$-SuperHyper Heptapartitioned Neutrosophic Set (n-SHHNS)* is a mapping

$$\widetilde{H}_n : \mathcal{P}_n(X) \rightarrow \mathcal{P}_n\big([0,1]^7\big),$$

such that for each $A \in \mathcal{P}_n(X)$,

$$\widetilde{H}_n(A) \subseteq \Big\{ (T, C, R, U, F, G, L) \in [0,1]^7 : T + C + R + U + F + G + L \leq 7 \Big\}.$$

Hence, every $n$-th level subset $A$ is assigned a *set* of membership 7-tuples $(T, C, R, U, F, G, L) \in [0,1]^7$ with $T + C + R + U + F + G + L \leq 7$.

**Theorem 2.27.** *Every Hyper Heptapartitioned Neutrosophic Set is a special case of an n-SuperHyper Heptapartitioned Neutrosophic Set for* $n = 1$.

*Proof.* A *Hyper Heptapartitioned Neutrosophic Set (HHNS)* $\widetilde{H}$ is a mapping $X \to \mathcal{P}([0,1]^7)$ with $T + C + R + U + F + G + L \leq 7$. In Definition 2.26, set $n = 1$, so

$$\widetilde{H}_1 : \mathcal{P}_1(X) = \mathcal{P}(X) \ \to\ \mathcal{P}_1\big([0,1]^7\big) = \mathcal{P}\big([0,1]^7\big).$$

Define

$$\widetilde{H}_1(\{x\}) := \widetilde{H}(x), \quad \widetilde{H}_1(A) = \varnothing \quad (\text{for } A \neq \{x\}).$$

Hence, for singletons $A = \{x\} \subseteq X$, we recover precisely the membership sets $\widetilde{H}(x)$. Thus, every HHNS is embedded in an $n$-SHHNS with $n = 1$. □

**Theorem 2.28.** *Every n-SuperHyperneutrosophic Set is a special case of an n-SuperHyper Heptapartitioned Neutrosophic Set by ignoring four membership components (e.g.* $C = R = G = L = 0$*).*

*Proof.* An *n-SuperHyperneutrosophic Set (SHNS)* $\tilde{A}_n$ maps each $A \in \mathcal{P}_n(X)$ to subsets of $[0,1]^3$ with $(T, I, F)$ membership. In Definition 2.26, we assign subsets of $[0,1]^7$ with $(T, C, R, U, F, G, L)$, each summing to at most 7. If we require $C = R = G = L = 0$ and rename $U = I$, then $(T, I, F)$ in $[0,1]^3$ is identified with $(T, 0, 0, I, F, 0, 0)$ in $[0,1]^7$. So define

$$\widetilde{H}_n(A) = \Big\{\big(T, 0, 0, I, F, 0, 0\big) : (T, I, F) \in \tilde{A}_n(A)\Big\}.$$

Hence, ignoring four extra membership dimensions reverts us to $(T, I, F) \in [0,1]^3$. Therefore, an SHNS is included in $n$-SHHNS by collapsing the four additional components. □

### 2.5 m-Polar Hyperneutrosophic Set

An *m-Polar Hyperneutrosophic Set* extends the conventional neutrosophic framework by assigning $m$ distinct truth, indeterminacy, and falsity triplets to each element in a given universe. This model provides a comprehensive structure for handling complex and multidimensional uncertainties [67, 75, 78, 83, 95, 98, 99, 102]. Related concepts include the bipolar neutrosophic set [1, 24, 77, 118], bipolar fuzzy set [3, 4, 66], tripolar fuzzy set [89–91], and m-polar fuzzy set [18, 63, 92, 93], which address specific dimensions of uncertainty and vagueness.

The m-Polar Neutrosophic Set is further extended using the frameworks of Hyperneutrosophic Sets and SuperHyperneutrosophic Sets, allowing for even more flexible and detailed representations of complex systems.

**Definition 2.29** (m-Polar Neutrosophic Set)**.** (cf. [83, 95, 98, 99, 102]) Let $X$ be a universe of discourse and $m \geq 1$ represent the number of poles or criteria. An $m$-polar neutrosophic set $A$ is defined as:

$$A = \left\{\langle x, (T_A^{(k)}(x), I_A^{(k)}(x), F_A^{(k)}(x))\rangle \mid x \in X, k = 1, 2, \ldots, m\right\}, AbdelBasset2019CosineSM,$$

where:

- $T_A^{(k)}(x), I_A^{(k)}(x), F_A^{(k)}(x) \in [0,1]$,
- $T_A^{(k)}(x) + I_A^{(k)}(x) + F_A^{(k)}(x) \leq 3,\ \forall x \in X,\ k = 1, 2, \ldots, m.$

**Definition 2.30** (m-Polar Hyperneutrosophic Set (m-HNS))**.** Let $X$ be a non-empty set, and let $m \geq 1$. Consider the family $\mathcal{P}\big([0,1]^3\big)$ of all non-empty subsets of the unit cube $[0,1]^3$. An *m-Polar Hyperneutrosophic Set* $\widetilde{M}$ on $X$ is defined by a mapping

$$\widetilde{M} : X \times \{1, 2, \ldots, m\} \quad \to \quad \mathcal{P}([0,1]^3),$$

such that for each $(x, k)$:

$$\widetilde{M}(x, k) \quad \subseteq \quad \big\{(T, I, F) \in [0,1]^3 : T + I + F \leq 3\big\}.$$

Hence, each element $x \in X$ and each pole $k$ is assigned a *set* of possible triplets $(T, I, F)$.

**Theorem 2.31.** *Every m-Polar Neutrosophic Set is a special case of an m-Polar Hyperneutrosophic Set.*

*Proof.* An *m-Polar Neutrosophic Set (mPNS) A* assigns each $(x, k)$ exactly one triplet $\big(T_A^{(k)}(x), I_A^{(k)}(x), F_A^{(k)}(x)\big)$ with $T + I + F \le 3$. In Definition 2.30, we let each $(x, k)$ map to *a set* in $[0, 1]^3$. Define

$$\widetilde{M}(x, k) = \Big\{\big(T_A^{(k)}(x), I_A^{(k)}(x), F_A^{(k)}(x)\big)\Big\},$$

a singleton. The usual constraint $T + I + F \le 3$ holds. Hence, every mPNS is embedded in an m-Polar Hyperneutrosophic Set as a degenerate case (singleton membership for each $(x, k)$). □

**Theorem 2.32.** *Every HyperNeutrosophic Set is a special case of an m-Polar Hyperneutrosophic Set by setting $m = 1$ (only one pole).*

*Proof.* A *HyperNeutrosophic Set (HNS)* $\tilde{A}$ is a mapping $X \to \mathcal{P}([0, 1]^3)$. In Definition 2.30, we have $\widetilde{M} : X \times \{1, 2, \dots, m\} \to \mathcal{P}([0, 1]^3)$. If $m = 1$, then for each $x$ we define

$$\widetilde{M}(x, 1) = \tilde{A}(x).$$

Hence, ignoring the multiple poles (just $k = 1$) yields exactly a HyperNeutrosophic Set. Therefore, an HNS is a sub-case of an m-Polar Hyperneutrosophic Set with $m = 1$. □

**Definition 2.33** (m-Polar n-SuperHyperneutrosophic Set (m-SHNS))**.** Let $X$ be a non-empty set, $m \ge 1$ be the number of poles, and define recursively:

$$\mathcal{P}_1(X) = \mathcal{P}(X), \quad \mathcal{P}_k(X) = \mathcal{P}\big(\mathcal{P}_{k-1}(X)\big) \quad (k \ge 2).$$

Similarly, consider $\mathcal{P}_n([0, 1]^3)$ for $n$-nested subsets of $[0, 1]^3$. An *m-Polar n-SuperHyperneutrosophic Set* $\widetilde{M}_n$ is given by a mapping

$$\widetilde{M}_n : \mathcal{P}_n(X) \times \{1, 2, \dots, m\} \longrightarrow \mathcal{P}_n\big([0, 1]^3\big),$$

such that for each $(A, k) \in \mathcal{P}_n(X) \times \{1, \dots, m\}$:

$$\widetilde{M}_n(A, k) \subseteq \big\{(T, I, F) \in [0, 1]^3 : T + I + F \le 3\big\}.$$

Hence, each $n$-th level subset $A$ and each pole $k$ is assigned a *set* of membership triplets $(T, I, F) \in [0, 1]^3$.

**Theorem 2.34.** *Every m-Polar Hyperneutrosophic Set is a special case of an m-Polar n-SuperHyperneutrosophic Set for $n = 1$.*

*Proof.* An *m-Polar Hyperneutrosophic Set* $\widetilde{M}$ (Definition 2.30) is a mapping:

$$X \times \{1, \dots, m\} \to \mathcal{P}([0, 1]^3).$$

In Definition 2.33, for $n = 1$ we have $\widetilde{M}_1 : \mathcal{P}_1(X) \times \{1, \dots, m\} \to \mathcal{P}_1([0, 1]^3) = \mathcal{P}([0, 1]^3)$. We can embed $\widetilde{M}$ by defining:

$$\widetilde{M}_1(\{x\}, k) := \widetilde{M}(x, k), \quad \widetilde{M}_1(A, k) = \varnothing \quad \text{if } A \ne \{x\}.$$

Hence, for singletons $\{x\} \subset X$, we recover exactly the membership sets from $\widetilde{M}(x, k)$. The constraint $T+I+F \le 3$ remains identical. Therefore, $\widetilde{M}$ is included in $\widetilde{M}_1$, which is an m-Polar 1-SuperHyperneutrosophic Set. □

**Theorem 2.35.** *Every n-SuperHyperneutrosophic Set is a special case of an m-Polar n-SuperHyperneutrosophic Set by letting $m = 1$ (only one pole).*

*Proof.* An *n-SuperHyperneutrosophic Set (SHNS)* $\tilde{A}_n$ is a mapping $\mathcal{P}_n(X) \to \mathcal{P}_n([0, 1]^3)$. In Definition 2.33, we have $\widetilde{M}_n : \mathcal{P}_n(X) \times \{1, \dots, m\} \to \mathcal{P}_n([0, 1]^3)$. If $m = 1$, we define

$$\widetilde{M}_n(A, 1) = \tilde{A}_n(A), \quad \widetilde{M}_n(A, k) = \varnothing \quad \text{for } k \ne 1.$$

Hence, ignoring multiple poles (just $k = 1$) we recover the usual $n$-SuperHyperneutrosophic mapping from $\mathcal{P}_n(X)$ to $\mathcal{P}_n([0, 1]^3)$. Therefore, an SHNS is embedded in m-SHNS with $m = 1$. □

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## Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

## Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

## Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

## Disclaimer

This work presents theoretical concepts that have not yet undergone practical testing or validation. Future researchers are encouraged to apply and assess these ideas in empirical contexts. While every effort has been made to ensure accuracy and appropriate referencing, unintentional errors or omissions may still exist. Readers are advised to verify referenced materials on their own. The views and conclusions expressed here are the authors' own and do not necessarily reflect those of their affiliated organizations.

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